Confidence Intervals

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Statistics in experimental science

- We want to build models of reality
- We perform experiments or make observations to test models – that is, to falsify them and build better models.
- In psychology and neuroscience, most of the time the problem is simply to estimate parameters of simple models.
- Building up confidence intervals is the statistical method to estimate these parameters.
    - Parameter estimation can make sense only as long as models capture reality well enough.
        - Applying a Normal model to skewed distributions is a non-sense. The analysis of residuals is the first step to evaluate the goodness of any model, before any parameter estimation.
        - How to deal with outliers, transformation of data, the choice of variables to include in the models are all important scientific choices, which engage the responsibility of the researcher, not of the statistician.

Statistics in experimental science

- We need statistics because our models and our measures are not precise enough. In particular, we can't control all the parameters.
  - This lack of precision introduces randomness in measurements.
  - Statistic tools help us to evaluate the strength of our models in face of randomness.

- The parameters we want to estimate are called "population" values or "statistics". A statistic can be
  - a mean
  - a percentage
  - a correlation
  - a slope
  - …

Inferences based on sampling theory

- The sampling theory studies the relationship between a parent population and the samples drawn from that population
- Samples are considered as drawn *randomly*
- Inferential statistics is the reverse logic: we want to get some information about a population by probing a sample.
- “p-values” are computed using the sampling theory.
  - They indicate the probability of obtaining a given sample conditional to a model of the population.
  - They don’t allow any probability statement on the population parameters.
  - They don’t allow any inference (Bayes Theorem). The common usage of taking decisions about models based on p-values is a statistical buffoonery which has been detrimental to science for at least the past 50 years.

Inferences based on sampling theory

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- Inferential statistics is the reverse logic: we want to get some information about a population by probing a sample.

- “confidence intervals” are also computed using the sampling theory.
  - They indicate the range of possible parameter values for a population statistic, based on a sample – under certain conditions, and for an arbitrary Confidence Level (95%).
  - Precisely, a 95% Confidence Interval corresponds to a *procedure* to build an interval based on observations (a sample) that, under repeated, independent, random samples in identical conditions (i.i.d.) would contain the true population parameter (statistic) 95% of the time.

Inferences based on sampling theory

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  - Precisely, a 95% Confidence Interval corresponds to a *procedure* to build an interval based on observations (a sample) that, under repeated, independent, random samples in identical conditions (i.i.d.) would contain the true population parameter (statistic) 95% of the time.
  - A 95% CI ***DOES NOT*** indicate that you have 95% chance that the Population value is within your 95%CI: you still cannot make any *probability statement* on your population (Bayes theorem).
  - However, for well behaved models (like, when residuals are normally distributed), a 95% CI is equivalent to a Bayesian 95% Higher Density Interval obtained with flat priors: if you don’t have any information before running your experiment, a HDI indicates that you have a 95% likelihood that the population value is within your 95% HDI.
  - ***Therefore, you can say that the population value is very likely within your 95% CI, as long as your model describes your data well enough.***

Sampling theory

Population Ω: (N, m, σ)

N individuals, each individual has a characteristic x_i

Mean $m = \frac{1}{N}\sum_{i=1}^{N} x_i$

Standard deviation $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - m)^2}$

Samples (n, $\overline{x}, \sigma'$) of size n

$\overline{x_1}, \sigma'_1$ mean and standard deviation of characteristic x_i in sample 1

$\overline{x_2}, \sigma'_2$ mean and standard deviation of characteristic x_i in sample 2

…

Sampling distribution $\overline{X}(\mu_{\overline{X}}, \sigma_{\overline{X}})$

$\overline{x_1}, \overline{x_2}$, … constitute a **sampling distribution** of means

The sampling theory studies the relationship between a parent population and the samples drawn from that population

Samples are considered as drawn *randomly*

Sampling theory

If each individual is drawn randomly, X is a random variable

The law of X is uniform $X(\Omega) = \{ X_1, \dots, X_N\}$

$$p(X = X_s) = 1/N$$

$$E(X) = m$$

$$V(X) = \sigma^2$$

$\overline{x}_1, \overline{x}_2, \ldots$ means of characteristic x_i in each sample can be interpreted as realizations of a random variable $\overline{X}$ with $E(\overline{X}) = \mu_{\overline{X}}$ ($\overline{X}$ sampling distribution of means)

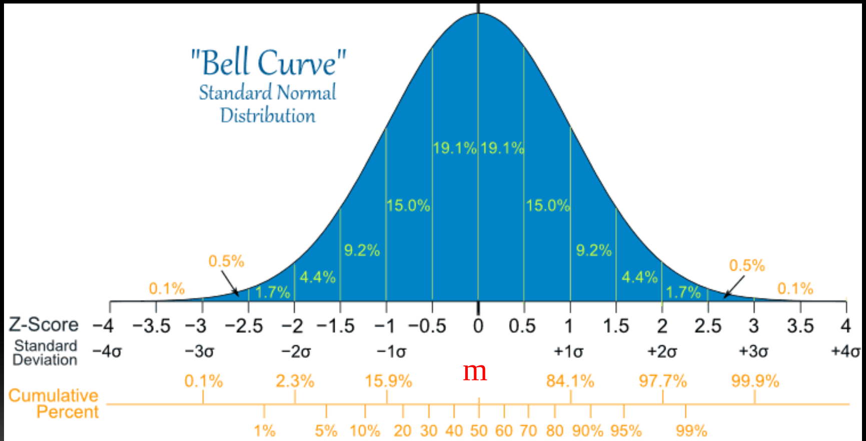
$\overline{x}, \sigma'$ in a sample are random variables

Theorem: the random variable $\overline{x}$ is defined by its

mean $E(\overline{x}) = m = \mu_{\overline{X}}$

standard deviation $S(\overline{x}) = \frac{\sigma}{\sqrt{n}} = \sigma_{\overline{X}}$

For non-exhaustive draws with replacements



Central Limit Theorem: for n > 30, the law of $\overline{X}$ *is asymptotically normal* $N(\mu_{\overline{X}}, \sigma_{\overline{X}}) = N(m, \frac{\sigma}{\sqrt{n}})$

Estimation

$\bar{x}$ unbiased estimator of m (and converging)

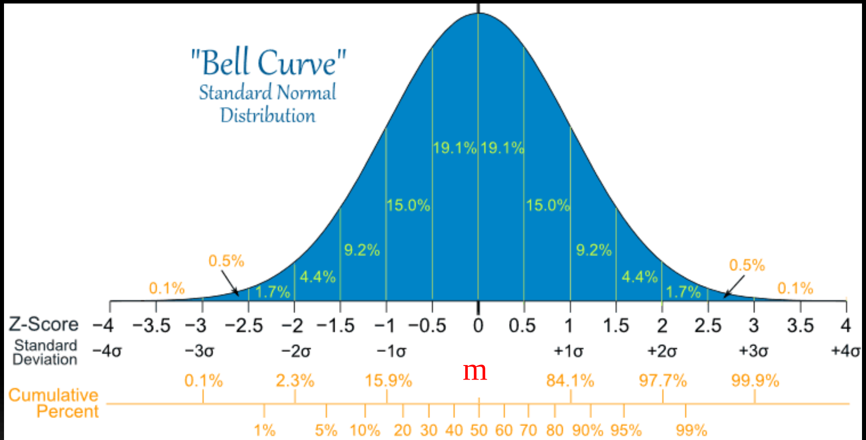
Estimator of population variance:

$s^2 = \frac{n}{n-1}\sigma'^2$ unbiased estimator of σ^2

($s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ *"unbiased sample variance"*)

Theorem: for n > 30 (# values in the sample), the law of $\bar{X}$ *can be estimated by the normal* $N(\mu_{\bar{X}}, \sigma_{\bar{X}}) \sim N(\bar{x}, \frac{s}{\sqrt{n}})$

Estimation of the distribution of Means of random samples:
95% of the Means of random samples (the sample statistics)
are comprised between $\bar{x}-1.96*\frac{s}{\sqrt{n}}$ and $\bar{x}+1.96*\frac{s}{\sqrt{n}}$

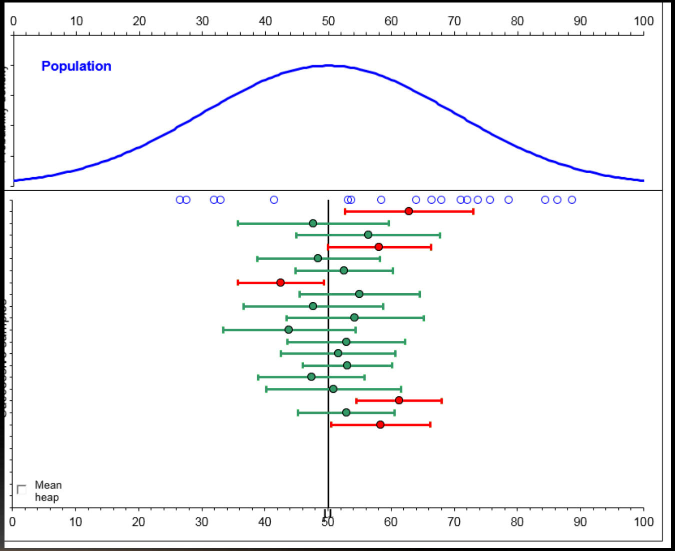


GENERAL FORMULA for a (1-alpha) CI of a Population parameter, based on a Sample statistic $\bar{x}$

w = z((1-alpha)/2) * Standard Error CI = [$\bar{x}$-w $\bar{x}$+w]

DEFINITION OF A CONFIDENCE INTERVAL

- If our ***model*** is correct
    - = one single population value with random noise: Gaussian model
        - check the residuals
        - similar variances if several groups
        - no outlier
- we can estimate the ***parameter*** of the model (the population value), and
- ***95% of the random samples will include the population value within their 95% CI***
    - Check it with simulations in R or Matlab
    - Or using Cumming's Excel macros:



http://www.latrobe.edu.au/psy/research/cognitive-and-developmental-psychology/esci
ESCI chapters 1-4 Jul 4 2011.xlsm, sheet CIjumping

For those who still care about p-values …

(“Null Hypothesis Significance Tests”, NHST)

THE SCIENTIFIC CONFIDENCE CRISIS

Why most published results are false in psychology and neuroscience

- “partout dans le monde, les centres de recherche semblaient de plus en plus avancer à l’aveuglette, dans un empirisme dénué de sens. Aucun résultat ne permettait d’approcher de la moindre conclusion, ni même de formuler la moindre hypothèse théorique”

  Michel Houellebecq, *Les particules élémentaires*, 1998

- Obvious responses
  - Science is produced by human beings
  - Humans are incompetent (data analysis), ambitious, lazy, dishonest. They have cognitive biases, they are believers. Logic runs against human cognitive habits.
- A structural culprit
  - NHST. Jacob Cohen (1994): ‘I make no pretense of the originality of my remarks […]. David Bakan said back in 1966 that his claim that “a great deal of mischief has been associated” with the test of significance “is hardly original”, that it is “what everybody knows” […] We, as teachers […] are responsible for the ritualization of NHST.’

“Significance tests as sorcery” (Lambdin 2012, quoting Andreski 1972)

- Test of significance as “statistical shamanism”: Many scientists make the claims they do empirical, real science, “not because they have corroborated, diverse evidence supporting them as accurate descriptors of reality, but rather because they desire their opinions to become reality”.
- "the continuing popularity of significance tests in our peer-reviewed journals is at best embarrassing and at worst intellectually dishonest”
- “The plain fact is that 70 years ago Ronald Fisher gave scientists a mathematical machine for turning baloney into breakthroughs, and flukes into funding. It is time to pull the plug.”

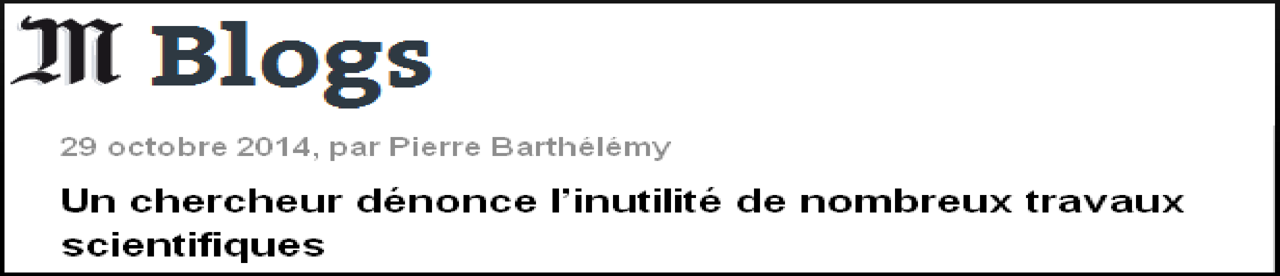
Robert Matthews, *Sunday Telegraph*, 1998

- The question today is not whether NHST are sound or not. They are not.

See the authoritative criticism of p-values by the American Statistical Association:

Ronald L. Wasserstein & Nicole A. Lazar (2016): The ASA’s statement on p-values: context, process, and purpose, The American Statistician, DOI:10.1080/00031305.2016.1154108.

The scientific confidence crisis



Blogs

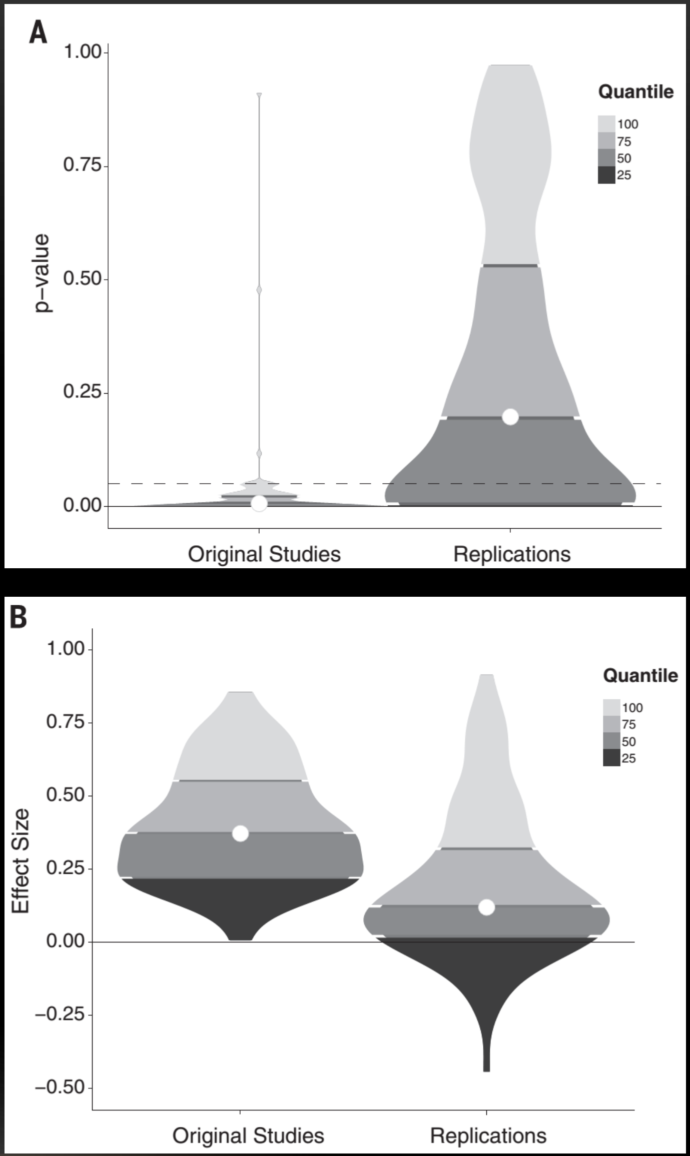
29 octobre 2014, par Pierre Barthélémy

Un chercheur dénonce l'inutilité de nombreux travaux scientifiques

- 'Currently, many published research findings are false or exaggerated, and an estimated 85% of research resources are wasted.' (Ioannidis 2014. *How to make more published research true*. PLoS Med)
  - Ioannidis (2005). *Why most published research findings are false*. PLoS Med
  - Button, Ioannidis, at al. (2013). *Power failure: why small sample size undermines the reliability of neuroscience*. Nat Rev Neurosci

Proven lack of reproducibility

- Thirty-six percent of replications had significant results.
- The mean effect size (r) of the replication effects (Mr = 0.197, SD = 0.257) was half the magnitude of the mean effect size of the original effects (Mr = 0.403, SD = 0.188)



Open Science Collaboration (2015) Estimating the reproducibility of psychological science. *Science*

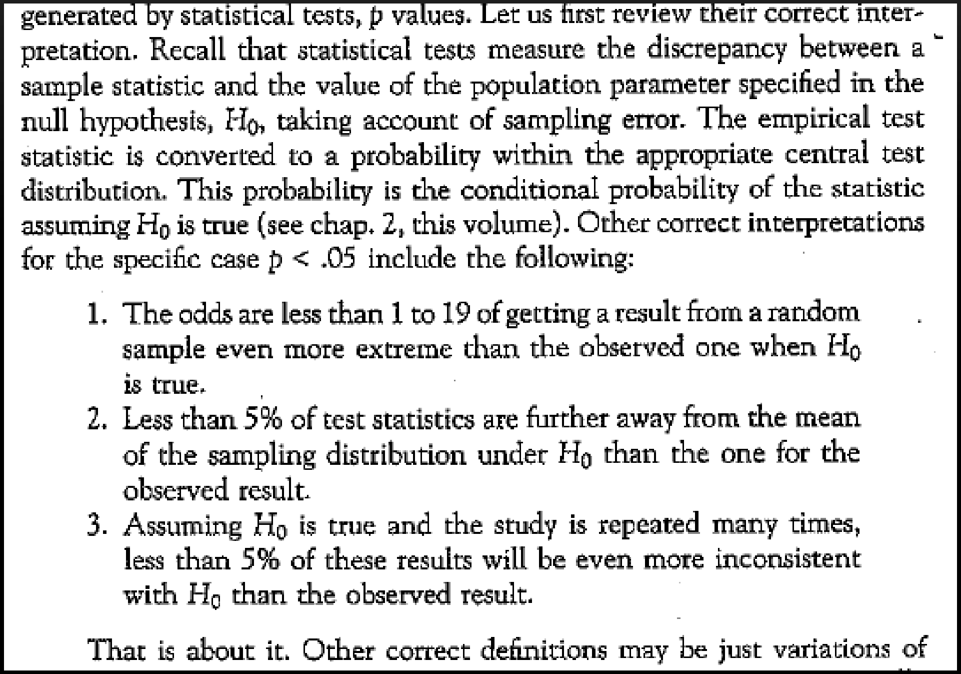
MORE DETAILED EXPLANATIONS OF P-VALUES (for those who still care about it …)

- The correct interpretation of a p-value

- “p < 0.05” (NHST)

  Example: acute treatment of heart attacks with streptokinase.
  Fictitious data: 4 deaths/20 with the drug, 10 deaths/20 with placebo
  p = 0.047 (H0 = no effect of the drug)

- if H0 is true less than 5% chance of observing at least such a large difference
- if H0 is true only 5% of samples should have a given statistics beyond *xxx (z>1.96)*
- What can I conclude ?
  - Fisher (1922-1935): “I’m surprised”, but I can’t make any probability statement on H0.
  - Neyman/Pearson (1933): *Decide* what is the most likely hypothesis (H0 or H1)
    - **Type I error**: to reject H0 when in fact it is true (but *the risk of this error is not p !!!*)
    - **Type II error**: to accept H0 when in fact it is false (*the risk of this error is not 1-p*)



generated by statistical tests, p values. Let us first review their correct interpretation. Recall that statistical tests measure the discrepancy between a sample statistic and the value of the population parameter specified in the null hypothesis, H_0, taking account of sampling error. The empirical test statistic is converted to a probability within the appropriate central test distribution. This probability is the conditional probability of the statistic assuming H_0 is true (see chap. 2, this volume). Other correct interpretations for the specific case $p < .05$ include the following:

1. The odds are less than 1 to 19 of getting a result from a random sample even more extreme than the observed one when H_0 is true.
2. Less than 5% of test statistics are further away from the mean of the sampling distribution under H_0 than the one for the observed result.
3. Assuming H_0 is true and the study is repeated many times, less than 5% of these results will be even more inconsistent with H_0 than the observed result.

That is about it. Other correct definitions may be just variations of

Kline (2004). Beyond Significance Testing: Reforming Data Analysis Methods in Behavioral Research. Washington, D.C.: APA Books.

What can we do with the p-values?

Manufacturing: Compute the cost of the guaranty and decide to take an insurance or not

contract = reimburse the client if > 5% bad pieces. Cost = *p-value* * Price

may be cheaper than having to check every piece

Science: test a theory (Popperian refutation)

e.g.: a study in genetics predicts that blue eyes should be found in 17.456 % of the population

I found 20%. Should I refute my theory?

I can compute the chances of observing my sample value if my theory is correct

- like in physics (I can never confirm my theory)

- but rarely the case in psychology: RT = 243 ms?

or neuroscience: BOLD increase = 0.57%?

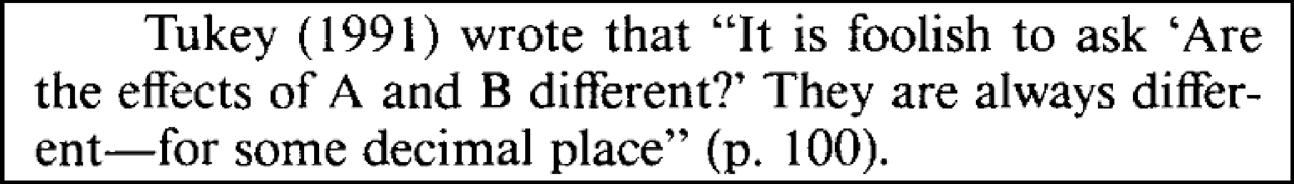
In psychology and neuroscience, we have almost no theory.

The only thing we can do is to try to *estimate* an RT or a BOLD increase

Meehl PE (1967) Theory-testing in psychology and physics: A methodological paradox. *Philosophy of Science*

Saving the p-value? Isn’t being surprised enough? (Popperian refutation)

- Specific predictions are rare in our field. Most of the time, the Null Hypothesis = 0 (“Nil” Hypothesis)
  - “Because the null is almost always false (Cohen,1994), investigators who imply that manipulations were effective on the basis of a p less than alpha are prone to Type III errors”
  - Type III error = correctly rejecting the Null hypothesis but for the wrong reason. False premises (the Nil hypothesis) “lead to conclusions that may be logically consistent but empirically invalid”.
  - Type III errors simply mean that it is almost impossible to compare two conditions (or populations) ‘everything else being equal’. The first thing to do is to perform good experiments with clever designs.



Tukey (1991) wrote that “It is foolish to ask ‘Are the effects of A and B different?’ They are always different—for some decimal place” (p. 100).

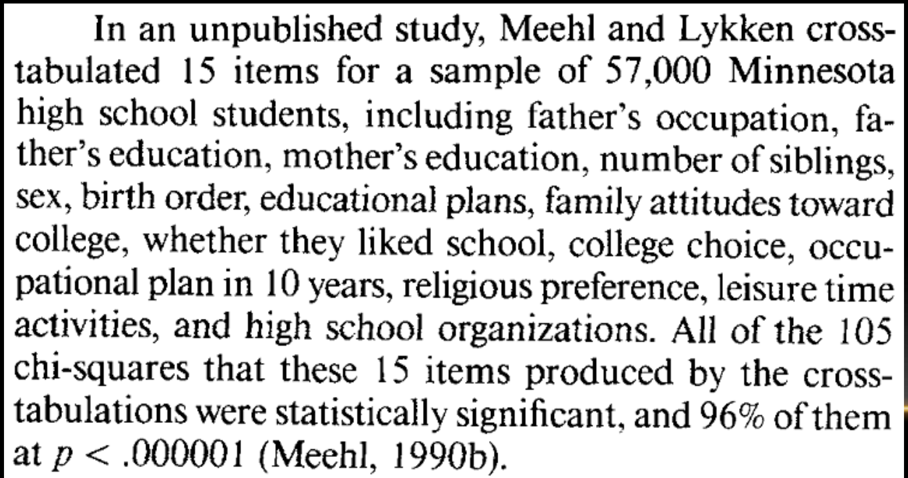
  - What we care about is how different: **effect sizes.**

Killeen PR (2005). An alternative to null hypothesis significance tests. *Psychological Science*

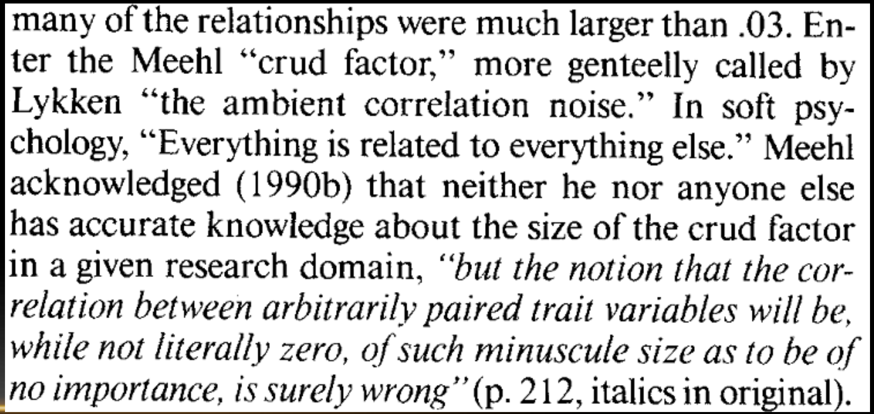
Cohen J (1994) The earth is round (p<.05). *American Psychologist*

Saving the p-value? Isn't being surprised enough? (Popperian refutation)

- H0 is always false, so effect sizes are never Null (d > 0)
  - Cohen's d = (m1-m2)/s
  - Student t-test = (m1-m2)/(s/sqrt(n))
  - t = d * sqrt(n)
- *with enough measures, any weak effect may be "significant"*. Weak effects are not reproducible, whatever the p-value



In an unpublished study, Meehl and Lykken cross-tabulated 15 items for a sample of 57,000 Minnesota high school students, including father's occupation, father's education, mother's education, number of siblings, sex, birth order, educational plans, family attitudes toward college, whether they liked school, college choice, occupational plan in 10 years, religious preference, leisure time activities, and high school organizations. All of the 105 chi-squares that these 15 items produced by the cross-tabulations were statistically significant, and 96% of them at $p < .000001$ (Meehl, 1990b).



many of the relationships were much larger than .03. Enter the Meehl "crud factor," more genteelly called by Lykken "the ambient correlation noise." In soft psychology, "Everything is related to everything else." Meehl acknowledged (1990b) that neither he nor anyone else has accurate knowledge about the size of the crud factor in a given research domain, *"but the notion that the correlation between arbitrarily paired trait variables will be, while not literally zero, of such minuscule size as to be of no importance, is surely wrong"* (p. 212, italics in original).

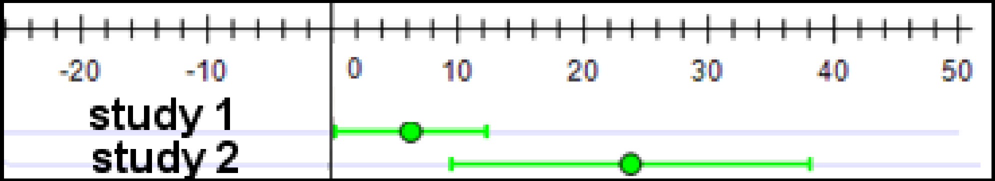
Cohen J (1994) The earth is round (p<.05). *American Psychologist*

- Do we simply need to request smaller p-values?
  - (equivalent to the 95% CI being further away from zero, so we are much more confident that there is a true effect)
- **NO**
  - *Because p-values do not indicate effect sizes. Small effect sizes are most of the time meaningless, even when "highly significant".*
  - *Because p-values indicate the probability of the sample CONDITIONAL to the Null hypothesis (which is always wrong, anyway: type III errors)*
    - Computations of the CIs for the population parameter are not always straightforward – the Normal case DOES NOT generalize (you need Bayesian simulations)
      - P-values obtained by **Bootstrap, Randomization (Permutation) or non-parametric tests** CANNOT be translated to *population CIs*!
        - Those tests (the way they are used now, to compute "*CIs" of the NULL hypothesis, confused with population CIs*) **are MOST OF THE TIME USELESS**

- **NHST do not allow us to reach any scientific conclusion**
  - On the one hand, "a test of significance contains no criterion for 'accepting' a hypothesis" (Fisher, 1959, p. 42), and on the other, we cannot safely reject a hypothesis without knowing the priors.
  - You may only be "surprised" by the value observed in your sample.
    - Is $p < 0.05$ a reason sufficient enough for being surprised? No, because H0 is always false!
      - at least when H0 = 0. If H0 is a specific value predicted by your theory, you may be surprised indeed. Useful to refute a theory, but certainly not to validate a theory.
- CIs do not allow any decision: 95% of your samples will include the true value within the 95% CI.
  - 'Don't look for a magic alternative to NHST, some other objective mechanical ritual to replace it. It does not exist'. (Cohen, 1994) (*Beware*: **Bayes factor** is an example of such a **magic alternative**).
  - We need to repeat experiments, wait for meta-analyses before risking making up stories. 'There is no free lunch in good research' (Ioannidis)
  - Small CI (PRECISION) are more informative
    - Publication of CI more difficult? Yes, at p ~= 0.05, CIs are "embarrassingly large" (Cohen, 1994)
    - But CIs do not make any difference between "null" and "positive" results.

Confidence Intervals: equivalent to p-values?

- Like a p-value, a CI does not allow any probability statement on the true value of the population
  - p-value: if H0 is true only 5% of samples should have a given statistics beyond *xxx (z>1.96)*
  - CI: a 95% CI means that 95% of your samples will include the true value within the 95% CI
- Example: 2 studies, p = .044 and p < .001



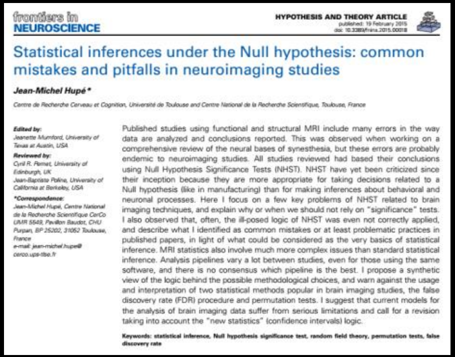
- At p = 0.05, the 95% CI includes (“touches”) the Null hypothesis

Region of Practical Equivalence (ROPE):
Cumulative probability around the null hypothesis (for example) instead of the point probability. One might decide that the Null hypothesis corresponds in fact to all effect sizes less than Cohen’s d = 0.2.
(or here, between – 20 and 20)
You may accept the Null hypothesis if the whole CI is included within the ROPE even if the Null is not included within the CI (high precision of small, practically « non-significant » or noise-based, effect).
p-values and CI logic lead to opposite conclusions about these 2 studies!

Summary

- You want an estimation of your effect size. The statistical summary of a study should be a CI of an effect size
    - “Error bars” : SEM or 95% CI? CI = 1.96 * SEM !! (for the normal distribution)
    - Need to show the CI of the difference even for between group comparisons
  - The smaller the CI (precision), the more informative is your study, because you are narrowing the range of hypotheses compatible with your data.
  - A small CI including 0 is more informative than a large CI not including zero (even if $p < 0.001$).
  - Reports of CI allow the publication of “Null” results and should avoid publication biases, selective reporting, file drawers problems and fraud.
- **What are the conditions of validity we should care about?**
  - For the estimation of the **mean**, the central limit theorem guaranties that when **n** is large enough, **the mean of the sample** converges to the **mean of the population**, *whatever its shape*, because what matters is only that the *distribution of sample means* is normal.
    - Some stat teachers conclude that when n > 30, you don’t even need to look at your distribution (correct statistical conclusion but totally stupid if you think about your model).
    - To build a CI, the most important information is the width, which depends on the estimation of the SD. The SD is critically dependent on asymmetry and outliers. No equivalent of the “central” limit theorem for SD (or is there?).

- **Must read short list, AVAILABLE AT http://cerco.ups-tlse.fr/-Charte-statistique-**
  - Meehl PE (1967) Theory-testing in psychology and physics: A methodological paradox. *Philosophy of Science* 34: 103–115.
  - Cohen J (1994) The earth is round (p<.05). *American Psychologist* 49: 997-1003
  - Kline (2004*). Beyond Significance Testing: Reforming Data Analysis Methods in Behavioral Research.* Washington, D.C.: APA Books.
  - Lambdin C (2012) Significance tests as sorcery: Science is empirical-significance tests are not. *Theory & Psychology* 22: 67–90.
  - Cumming G (2013) The new statistics: why and how. *Psychological Science* 25: 7-29.
    - Videos of Cumming's pedagogical tutorial (2014 APS Annual Convention in San Francisco): http://www.psychologicalscience.org/index.php/members/new-statistics
- Other recommended readings
  - Hupé 2015
  - http://dx.doi.org/10.3389/fnins.2015.00018



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Statistical inferences under the Null hypothesis: common mistakes and pitfalls in neuroimaging studies

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Published studies using functional and structural MRI include many errors in the way data are analyzed and conclusions reported. This was observed when working on a comprehensive review of the neural bases of synesthesia, but these errors are probably endemic to neuroimaging studies. All studies reviewed had based their conclusions using Null Hypothesis Significance Tests (NHST). NHST have yet been criticized since their inception because they are more appropriate for taking decisions related to a Null hypothesis (like in manufacturing) than for making inferences about behavioral and neuronal processes. Here I focus on a few key problems of NHST related to brain imaging techniques, and explain why or when we should not rely on "significance" tests. I also observed that, often, the ill-posed logic of NHST was even not correctly applied, and describe what I identified as common mistakes or at least problematic practices in published papers, in light of what could be considered as the very basics of statistical inference. MRI statistics also involve much more complex issues than standard statistical inference. Analysis pipelines vary a lot between studies, even for those using the same software, and there is no consensus which pipeline is the best. I propose a synthetic view of the logic behind the possible methodological choices, and warn against the usage and interpretation of two statistical methods popular in brain imaging studies, the false discovery rate (FDR) procedure and permutation tests. I suggest that current models for the analysis of brain imaging data suffer from serious limitations and call for a revision taking into account the "new statistics" (confidence intervals) logic.

Keywords: statistical inference, Null hypothesis significance test, random field theory, permutation tests, false discovery rate

- Examples of uses of CI, to present your data, for their interpretation, and to cite the literature:
  - Ruiz & Hupé 2015
  - **http://journals.plos.org/plosone/article?id=10.1371/journal.pone.01193770119377**
  - Chun & Hupé 2015
  - **http://onlinelibrary.wiley.com/doi/10.1111/bjop.12146/abstract**